

HERWIRI2: CEEX Electroweak Corrections in a Hadronic MC

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Reaching the 1% precision level for W and Z production calculations for the LHC will require a mixture of higher order QCD and electroweak corrections. As a first step toward implementing the combined QED \otimes QCD exponentiation proposed in previous work, we have implemented the $\mathcal{O}(\alpha)$ electroweak corrections and YFS exponentiation structure of the KK Monte Carlo in HERWIG. We discuss the current status of this program and sketch the further developments needed to reach the desired precision level.

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Vector boson production is one of the most important Standard Model processes observed at the LHC, and electroweak radiative corrections will be needed for analysis at the percent level. Previous studies [1] by some of the authors have found that electroweak corrections alone can exceed 1% for some cuts of interest. These studies were based on HORACE, [2] which provides state-of-the-art $\mathcal{O}(\alpha)$ radiative corrections with a final-state photon shower, and PHOTOS, [3] which adds final state photonic radiation. Other programs developed for implementing electroweak corrections for hadronic collisions include WINHAC [4] and ZINHAC[5]. Initial state photonic radiation has been considered only in certain MRST parton distribution functions. [6] However, none of the most recent PDFs include QED effects.

For electron-positron colliders, precision electroweak corrections have been implemented in the program *KKMC* [7] using YFS [8] exponentiated multiple-photon radiation for both the initial and final state, together with $\mathcal{O}(\alpha)$ electroweak corrections in the DIZET [9] package developed for ZFITTER[10]. The DIZET corrections can be applied to any parton-level process, so there is no obstacle to extending them to hadronic collisions. YFS exponentiation provides the basis for an efficient representation of multi-photon phase space, which can also be implemented for hadronic initial and final states. Some of the authors have proposed using a non-abelian extension of YFS exponentiation as a basis for an integrated event generator implementing both multi-photon and multi-gluon corrections in a unified framework, called $\text{QCD} \otimes \text{QED}$ exponentiation. [11]

The collection of programs implementing $\text{QCD} \otimes \text{QED}$ exponentiation has been called HERWIRI, for High Energy Radiation with Infra-Red Improvements, with a version number distinguishing the class of corrections included. The name acknowledges that the initial versions build upon the HERWIG [12] parton shower generator. The first to be released, HERWIRI1, [13] implemented IR-improved splitting kernels [14] obtained using the QCD analog of YFS exponentiation. This program is publicly available, and tests are in progress. The IR-improved kernels have also been implemented [15] in MC@NLO [16] and POWHEG[17]. The structure of HERWIRI is not tied to a particular shower, and our ultimate goal is a complete shower generator based entirely on $\text{QCD} \otimes \text{QED}$ exponentiation with exact $\mathcal{O}(\alpha_s^2, \alpha_s \alpha, \alpha^2)$ residuals. [18]

The second version, HERWIRI2, implements the electroweak radiative corrections of *KKMC* in a hadronic shower generator. This note describes a version of HERWIRI2 which is presently nearing completion.¹ HERWIRI2 is motivated by the successful application of YFS exponentiation in BHLUMI[19], BHWIDE[20], KoralZ[21], KoralW[22], YFSWW3[23], *KKMC*, and related programs for LEP physics. All of these programs benefit from a very efficient representation of N -photon phase space, with complete control over the soft and collinear singularities for an arbitrary number of photons. Real and virtual IR singularities cancel exactly to all orders. The non-abelian extension to $\text{QCD} \otimes \text{QED}$ should have similar advantages.

While based on HERWIG, HERWIRI2 is largely independent of the underlying shower. HERWIG generates the parton momenta and shower, and HERWIRI2 passes the generated hard process momenta to *KKMC* routines to add photons and electroweak corrections. Although *KKMC* was developed for e^+e^- collisions, it was designed to be extended to more general processes, so the ability to select quarks as the incoming state already exists in all but the lowest-level generation routines.

¹HERWIRI2 does not incorporate HERWIRI1, although the two programs can be used in combination.

KKMC is a precision generator for $e^+e^- \rightarrow f\bar{f} + n\gamma$, $f = \mu, \tau, d, u, s, c, b$ for CMS energies from $2m_\tau$ to 1 TeV. The precision tag for LEP2 was 0.2%. ISR and FSR γ emission is calculated up to $\mathcal{O}(\alpha^2)$, including interference. The MC structure is based on YFS exponentiation, including residuals calculated perturbatively to the relevant orders in $\alpha^k L^l$. ($L = \ln(s/m_e^2)$). Exact collinear bremsstrahlung is implemented for up to three photons. Electroweak corrections [24, 25, 26] are included via DIZET 6.21[9], and beamstrahlung can be modeled over a wide range of energies via a built-in or user-defined distribution.

There are two modes of operation: exclusive exponentiation (EEX) and coherent exclusive exponentiation (CEEEX). EEX applies YFS exponentiation to differential cross-sections, while CEEEX applies it at the amplitude level. The CEEEX mode is written in a manner that is most readily extended to quark scattering, so it is taken as the basis for HERWIRI2. The orders of residuals included in CEEEX mode are $\alpha, \alpha L, \alpha^2 L^2$, and $\alpha^2 L$.

CEEEX was introduced for pragmatic reasons, because the traditional (EEX) exponentiation of spin-summed differential cross sections suffered from a proliferation of interference terms in processes with multiple diagrams, limiting its utility in reaching the desired 0.2% precision tag for LEP2.[27] CEEEX works at the level of spinor helicity amplitudes, greatly facilitating the calculation of effects such as ISR-FSR interference, which are included in *KKMC*, and therefore in HERWIRI2. CEEEX is maximally inclusive: all real photons radiated are kept in the event record, no matter how soft or collinear. There is no need to integrate out a region of soft phase space, because the exponentiated amplitudes are well-behaved at $k = 0$.

The CEEEX cross section for $q\bar{q} \rightarrow f\bar{f}$ has the form [7]

$$\sigma = \frac{1}{\text{flux}} \sum_{n=0}^{\infty} \int d\text{PS} \rho_{\text{CEEEX}}^{(n)}(\vec{p}, \vec{k}) \quad (1)$$

where

$$\rho_{\text{CEEEX}}^{(n)} = \frac{1}{n!} e^{Y(\vec{p}, E_{\min})} \frac{1}{4} \sum_{\vec{\lambda}, \vec{\mu}} \left| \mathcal{M} \left(\begin{array}{cc} \vec{p} & \vec{k} \\ \vec{\lambda} & \vec{\mu} \end{array} \right) \right|^2 \quad (2)$$

The YFS form factor is

$$Y(\vec{p}, E_{\min}) = Q_i^2 Y(p_1, p_2, E_{\min}) + Q_f^2 Y(p_3, p_4, E_{\min}) + Q_i Q_f Y(p_1, p_3, E_{\min}) \\ + Q_i Q_f Y(p_2, p_4, E_{\min}) - Q_i Q_f Y(p_1, p_4, E_{\min}) - Q_i Q_f Y(p_2, p_3, E_{\min}), \quad (3)$$

where Q_i, Q_f are initial and final parton charges, and

$$Y(p_i, p_j, E_{\min}) = 2\alpha \tilde{B}(p_i, p_j, E_{\min}) + 2\alpha \text{Re } B(p_i, p_j), \quad (4)$$

with real and virtual form factors defined respectively by

$$\tilde{B} = - \int_{k^0 < E_{\min}} \frac{d^3 k}{8\pi^2 k^0} \left(\frac{p_i}{p_i \cdot k} - \frac{p_j}{p_j \cdot k} \right)^2, \quad (5)$$

$$B = \frac{i}{(2\pi)^3} \int \frac{d^4 k}{k^2} \left(\frac{2p_i + k}{2p_i \cdot k + k^2} - \frac{2p_j - k}{2p_j \cdot k - k^2} \right). \quad (6)$$

The n -photon helicity-spinor amplitude can be expanded in terms of order α^r having the form

$$\mathcal{M}_n^{(r)} = \sum_{\mathcal{P}} \prod_{i=1}^n \mathcal{S}_i^{(\mathcal{P}_i)} \left[\beta_0^{(r)} \left(\vec{p}; \vec{\lambda}; X_{\mathcal{P}} \right) + \sum_{j=1}^n \frac{\beta_1^{(r)} \left(\vec{p} \begin{smallmatrix} k \\ \vec{\lambda} \end{smallmatrix} \mu; X_{\mathcal{P}} \right)}{\mathcal{S}_j^{(\mathcal{P}_j)}} + \cdots + \sum_{1 < j_1 < \cdots < j_n} \frac{\beta_n^{(r)} \left(\vec{p} \begin{smallmatrix} \vec{k} \\ \vec{\lambda} \end{smallmatrix} \vec{\mu}; X_{\mathcal{P}} \right)}{\mathcal{S}_{j_1}^{(\mathcal{P}_{j_1})} \cdots \mathcal{S}_{j_n}^{(\mathcal{P}_{j_n})}} \right] \quad (7)$$

with residual spinor amplitudes $\beta_i^{(r)}$ and complex soft photon factors \mathcal{S}_j with the property

$$|\mathcal{S}_j^{(\mathcal{P}_j)}| = -2\pi\alpha Q^2 \left(\frac{p_a}{p_a \cdot k_j} - \frac{p_b}{p_b \cdot k_j} \right)^2 \quad (8)$$

where Q, p_a, p_b belong to the initial or final fermions depending on the partition \mathcal{P}_j .

KKMC incorporates the DIZET library (version 6.21) from the program ZFITTER. [10] The γ and Z propagators are multiplied by vacuum polarization factors:

$$H_\gamma = \frac{1}{2 - \Pi_\gamma}, \quad H_Z = 4 \sin^2(2\theta_W) \frac{\rho_{EW} G_\mu M_Z^2}{8\pi\alpha\sqrt{2}}. \quad (9)$$

Vertex corrections are incorporated into the coupling of Z to f via form factors in the vector coupling:

$$g_V^{(Z,f)} = \frac{T_3^{(f)}}{\sin(2\theta_W)} - Q_f F_V^{(f)}(s) \tan \theta_W. \quad (10)$$

Box diagrams contain these plus a new angle-dependent form-factor in the doubly-vector component:

$$g_V^{(Z,i)} g_V^{(Z,f)} = \frac{T_3^{(i)} T_3^{(f)} - 2T_3^{(i)} Q_f F_V^{(f)}(s) - 2Q_i T_3^{(f)} F_V^{(i)}(s) + 4Q_i Q_f F_{\text{box}}^{(i,f)}(s, t)}{\sin^2(2\theta_W)}. \quad (11)$$

The correction factors are calculated at the beginning of a run and stored in tables.

The Drell-Yan cross section with multiple-photon emission can be expressed as an integral over the parton-level process $q_i(p_1)\bar{q}_i(p_2) \rightarrow f(p_3)\bar{f}(p_4) + n\gamma(k)$, integrated over phase space and summed over photons. The parton momenta p_1, p_2 are generated using parton distribution functions giving a process at CMS energy q and momentum fractions x_1, x_2 such that $q^2 = x_1 x_2 s$:

$$\sigma_{\text{DY}} = \int \frac{dx_1}{x_1} \frac{dx_2}{x_2} \sum_i f_i(q, x_1) \bar{f}_i(q, x_2) \sigma_i(q^2) \delta(q^2 - x_1 x_2 s), \quad (12)$$

where the final state phase space includes p_3, p_4 and $k_i, i = 1, \dots, n$ and multiple gluon radiation + hadronization is included through a shower.

The parton-level cross section $\sigma_i(q^2)$ can be calculated by KKMC, which integrates over a final space phase space with two fermions and an arbitrary number of photons:

$$\sigma_i(q^2) = \sum_{n=0}^{\infty} \int d\text{PS}_{2+n} \sigma_i(\vec{p}, \vec{k}) \quad (13)$$

HERWIRI2 uses HERWIG 6.5 as the shower generator, which creates the hard process first, at Born level, in subroutine HWEPRO (HWHYP), and then passes it to the cascade generator HWBGEN. HERWIRI2 finds the Z/γ^* and the partons interacting with it in the event record. The

initial partons define p_1, p_2 , which are transformed to the CM frame and projected on-shell to create a starting point for *KKMC*, which generates the final fermion momenta p_3, p_4 and photons k_i (both ISR and FSR.) The generated particles are transformed back to the lab frame and placed in the event record.

In addition to the basic DY process, HERWIG generates “Compton” events $g + q \rightarrow q + Z/\gamma^*$. About 10% of the events have this form. This is factorized into gluon emission times a hard EW process at a shifted value of q^2 . These have a different profile in the event record, but can be processed by *KKMC* as well. There is also a third class of events with the emission of an additional hard gluon. About 1% of the events have this form, and also have a significant shift of the Z energy from its generation scale.

With a change of variables, the Drell-Yan cross section in HERWIG can be expressed as

$$\begin{aligned}\sigma_{\text{DY}} &= \int \frac{dx_1}{x_1} \frac{dx_2}{x_2} \sum_i f_i(q, x_1) f_{\bar{i}}(q, x_2) \sigma_i(q^2) \delta(q^2 - x_1 x_2 s) \\ &= \int_{q_{\min}}^{q_{\max}} dq P(q) \int_{q^2/s}^1 \frac{dx_1}{x_1} \sum_i P_i W_{\text{HW}}^{(i)}(q^2, x_1) = \langle W_{\text{HW}} \rangle\end{aligned}\quad (14)$$

where $P(q)$ is a normalized, integrable, crude probability distribution for q , P_i is the crude probability of generating parton i , and W_{HW} is the HERWIG event weight. This weight depends only on the hard Born cross section and is not altered by the shower.

The crude probability distributions used by HERWIG are

$$P(q) = \frac{1}{2} [P_\gamma(q) + P_Z(q)], \quad P_\gamma(q) = \frac{N_\gamma}{q^4}, \quad P_Z(q) = \frac{N_Z q}{(q^2 - M_Z^2) + \Gamma_Z^2 M_Z^2} \quad (15)$$

The HERWIG event weight is

$$W_{\text{HW}} = \sum_i W_{\text{HW}}^{(i)}, \quad W_{\text{HW}}^{(i)} = \frac{1}{P(q)} f_i(q, x_1) f_{\bar{i}}(q, x_2) \ln\left(\frac{s}{q^2}\right) \sigma_{\text{HW}}^{(i)}(q^2) \quad (16)$$

and the corresponding probability for selecting parton i is

$$P_i = W_{\text{HW}}^{(i)} / W_{\text{HW}} \quad (17)$$

We have chosen to introduce electroweak corrections in a minimally invasive way, incorporating them in a form factor

$$F_{\text{EW}}^{(i)}(q^2) = \frac{\sigma_i(q^2)}{\sigma_{\text{Born}}^{(i)}(q^2)} \quad (18)$$

KKMC will calculate the EW form factor, and multiply it by the HERWIG Born cross section.

$$\sigma_{\text{HW+EW}} = \langle W_{\text{Tot}} \rangle, \quad W_{\text{Tot}} = F_{\text{EW}}^{(i)}(q^2) W_{\text{HW}} = W_{\text{HW}} \frac{\sigma_{\text{KK}}^{(i)}(q^2)}{\sigma_{\text{Born}}^{(i)}(q^2)}. \quad (19)$$

The *KKMC* cross section is calculated using a primary distribution

$$\frac{d\sigma_{\text{Pr}}^{(i)}(s, v)}{dv} = \sigma_{\text{Born}}^{(i)}(s(1-v)) \frac{1}{2} \left(1 + \frac{1}{\sqrt{1-v}}\right) \bar{\gamma}_i v^{\bar{\gamma}_i-1} v_{\min}^{\gamma_i-\bar{\gamma}_i} \quad (20)$$

with

$$\gamma_i = \frac{2\alpha}{\pi} Q_i^2 \left[\ln \left(\frac{s}{m_i^2} \right) - 1 \right], \quad \bar{\gamma}_i = \frac{2\alpha}{\pi} Q_i^2 \ln \left(\frac{s}{m_i^2} \right) \quad (21)$$

to generate the factor v giving the fraction of s remaining after ISR photon emission, $s_X = s(1 - v)$.

The *KKMC* cross section is

$$\sigma(q^2) = \int d\sigma_{\text{Pri}} \frac{d\sigma_{\text{Cru}}}{d\sigma_{\text{Pri}}} \frac{d\sigma_{\text{Mod}}}{d\sigma_{\text{Cru}}} = \sigma_{\text{Pri}} \langle W_{\text{Cru}} W_{\text{Mod}} \rangle. \quad (22)$$

W_{Cru} is calculated during ISR generation and W_{Mod} is generated after s_X is available.

The *HERWIG* and *KKMC* weights are combined to calculate the total *HERWIRI2* weight,

$$\sigma_{\text{Tot}} = \left\langle W_{\text{HW}} \frac{\sigma_i(q^2)}{\sigma_{\text{Born}}^{(i)*}(q^2)} \right\rangle = \left\langle W_{\text{HW}} \sigma_{\text{Pri}}^{(i)}(q^2) \frac{W_{\text{Cru}}^{(i)} W_{\text{Mod}}^{(i)}}{\sigma_{\text{Born}}^{(i)*}(q^2)} \right\rangle, \quad (23)$$

This average *could* be calculated using a joint probability distribution for q and v , $D(q, v) = P(q) d\sigma_{\text{Pri}}/dv$, with $P(q)$ from *HERWIG*. An adaptive MC (S. Jadach's FOAM [28]) could calculate the normalization of the distribution at the beginning of the run, in a similar manner to how *KKMC* presently integrates the one-dimensional primary distribution. To account for beamsstrahlung, *KKMC* already permits such a user-defined distribution, in up to three variables. However, as a first step, we have tried to run *HERWIRI2* using *KKMC*'s one-dimensional primary distribution. This requires fixing an overall scale q_0 to initialize *KKMC* (e.g., $q_0 = M_Z$).

The built-in primary distribution for electrons at scale q_0 can be used for the low-level generation of v . The transformation from this distribution to a distribution at *HERWIG*'s generated scale q for quark i is then obtained by a change of variables:

$$\sigma_{\text{Tot}} = \sigma_{\text{Pri}}^{(e)} \left\langle W_{\text{HW}} \left(\frac{d\sigma_{\text{Pri}}^{(i)}(q^2, v)}{d\sigma_{\text{Pri}}^{(e)}(q_0^2, v)} \right) \left(\frac{W_{\text{Cru}}^{(i)} W_{\text{Mod}}^{(i)}}{\sigma_{\text{Born}}^{(i)*}(q^2)} \right) \right\rangle \quad (24)$$

with

$$\frac{d\sigma_{\text{Pri}}^{(i)}(q^2, v)}{d\sigma_{\text{Pri}}^{(e)}(q_0^2, v)} = W_\gamma^{(i)} \frac{\sigma_{\text{Born}}^{(i)}(q^2(1 - v))}{\sigma_{\text{Born}}^{(e)}(q_0^2(1 - v))}, \quad (25)$$

where

$$W_\gamma = \frac{\bar{\gamma}_i}{\bar{\gamma}_e} \left(\frac{v}{v_{\text{min}}} \right)^{\bar{\gamma}_i - \bar{\gamma}_e} v_{\text{min}}^{\bar{\gamma}_i - \bar{\gamma}_e}. \quad (26)$$

The γ factors are calculated using q^2/m_i^2 for parton i and q_0^2/m_e^2 for the electron.

Shuffling the numerators and denominators about gives the expression used in *HERWIRI2*:

$$\sigma_{\text{Tot}} = \langle W_{\text{HW}} W_{\text{Mod}} W_{\text{Karl}} W_{\text{FF}} W_\gamma \rangle \quad (27)$$

with two new weights

$$W_{\text{Karl}} = \frac{\sigma_{\text{Pri}}^{(e)} W_{\text{Cru}}^{(i)}}{\sigma_{\text{Born}}^{(e)}(q_0^2(1 - v))}, \quad W_{\text{FF}} = \frac{\sigma_{\text{Born}}^{(i)}(q^2(1 - v))}{\sigma_{\text{Born}}^{(i)*}(q^2)}. \quad (28)$$

HERWIRI2 is still under development, so any numerical results must be treated as preliminary. A run for pp collisions at 10 TeV with the Z/γ^* invariant mass bounded by 40 GeV and 140

GeV, using HERWIG 6.520 default parameters and PDFs, yields a cross-section of 1183.7 ± 1.3 pb, compared to 1098.8 ± 1.0 pb for HERWIG alone, giving an electroweak contribution of 7.7%. Turning on ISR gives a much wider weight distribution and consequently, greatly reduced efficiency. Preliminary results give a cross-section of 1212 ± 109 pb, showing an additional 2.4% contribution from ISR.

Work is in progress to optimize MC generation in the presence of ISR. Once HERWIRI2 is complete and thoroughly tested, it will be compared to other available hadronic/electroweak generators. In particular, it will be interesting to see the effect of initial state radiation, which is not present in the other programs, but appears to enter at the 2 – 3% level, making it crucial to precision calculations.

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